Numerical analysis of highly iterated fractal antennas

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Abstract: Analysis of highly iterated fractal antennas using integral equation methods is addressed in this paper. Time and memory requirements are greatly reduced taking advantage of the recursive structure of the fractal geometry.

1. Introduction

From the point of view of antennas, fractal structures based on an iterated function system (IFS) [1] are particularly interesting due to their multiband behavior [2], [3]. In addition, the highly convoluted shape of these geometries as the number of fractal iterations increases makes possible the reduction in size of certain antennas [4].

If only few fractal iterations are considered the well-known integral equation method discretized by Method of Moments (MoM) [5] can be easily used to compute numerically the radiation parameters of the antenna. However, when trying to predict the behavior of a highly iterated fractal antenna many small geometry details appear, so very small MoM subdomain functions are required to obtain an accurate discretization of the induced current. This results in a very large number of unknowns \(N\), and the computational requirements to solve the full linear system using conventional methods (memory increases as \(N^2\) and CPU time as \(N^3\)) can easily overcome the capabilities of desktop computer systems.

In this paper, the geometrical properties of the IFS that generates the antenna geometry are used in order to analyze a highly iterated fractal antenna with reasonable computational requirements. Several numerical considerations about the MoM linear system to solve will be made and used to analyze a Sierpinski triangular patch with several fractal iterations.

2. Numerical considerations

The electric field integral equation (EFIE) in the frequency domain discretized by MoM may be expressed in matrix form as [5]

\[- [E_j] = [Z] \cdot [J] \]  

(1)
where \([J]\) are the coefficients of the discretized induced current (unknowns), \([E_i]\) is the discretization of the incident field and \([Z]\) the impedance matrix which contains the Green’s function with information about the medium.

The IFS that generates the fractal consists on a set of affine transformations such as scale factors, translations or rotations, so if the Green’s function has any of these symmetries (i.e. translation symmetry) the impedance matrix has plenty of redundant information. This can be seen depicting \([Z]\) as an image if a minimum of care is taken when numbering the basis and test functions (Fig. 1). Since there are many sets of equal submatrices in \([Z]\) at different levels (specially in the diagonal of the matrix), an important saving of time an memory can be achieved avoiding the computation and storage of redundant matrix elements.

Once \([Z]\) has been built, the linear system (1) can be solved either by direct matrix inversion or by using an iterative method [6]. Here the question is if we can reduce the computational requirements making use of the redundant information.

For the case of direct matrix inversion, it can be seen that although the elements of \([Z]^{-1}\) have also a distribution in blocks similar to \([Z]\), each diagonal block is slightly different from the other diagonal blocks (Fig. 2). As a first approximation these blocks could be assumed to be equal to save memory and time, but when the number of iterations of the fractal increases \([Z]\) becomes ill-conditioned, specially for low frequencies, and small errors in \([Z]^{-1}\) result in large errors in the final solution. Then, as far as all the elements of \([Z]^{-1}\) must be obtained the computation time will grow as \(N^3\), making this option unaffordable for a large number of unknowns.

The coefficients of \([J]\) can be also found using an iterative method like the Generalized Minimum Residual (GMRES) [6]. In each iteration, the main computational effort to obtain the \(k\)th estimation of the induced current \([J^{(k)}]\) are the matrix vector products \([Z][J^{(k-1)}]\). Using direct matrix-vector multiplication, the operation count and the memory requirements for each iteration are proportional to \(N^2\). The operation count can be reduced to \(N\log N\) if the Multilevel Matrix Decomposition Algorithm (MLMDA) [7], [8] is used. This method performs a multilevel subdivision of the object that fits perfectly with the IFS building blocks, leading to a dramatic reduction in computation time and memory storage.

3. Analysis of a Sierpinski triangular patch

As example of the computational savings that can be achieved let us consider a microstrip Sierpinski patch antenna 8.89 cm height over a dielectric substract 1.57 mm thick with a relative permittivity of 2.33. The patch is excited with a standard coaxial probe located in the lower corner of the Sierpinski fractal. In
order to evaluate the behavior of the structure as the number of fractal iterations increases configurations with four, five, six and seven iterations have been analyzed.

The computer used in the simulation is a desktop PC with an AMD Athlon CPU at 1.33 GHz and 1.5 GB of RAM. The programming language is MATLAB 6 with time-critical routines coded in C. The differences in the computational requirements of the classic iterative solution in table I and the optimized algorithm in table II are impressive, specially as the number of fractal iterations increases. Now, the preconditioning step, which has not been optimized for fractals yet, seems to be the next bottleneck to solve.

4. Conclusions

Fractal structures built using the IFS concept have a multilevel structure and redundant information. This can be used to reduce dramatically the computational requirements in the numerical analysis of highly iterated fractals.

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References

Table II: MoM+GMRES+MLMDA optimized, \( f = 9 \) GHz.

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<th>Iteration</th>
<th>Mem in MB</th>
<th>Mem t</th>
<th>Mem t</th>
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<th>txit</th>
<th>It.time</th>
<th>Mem. time</th>
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<td>5.3 5</td>
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<td>9 2.60</td>
<td>23.4</td>
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<td>166.5 176</td>
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Table I: MoM+GMRES, \( f = 9 \) GHz.

Table II: MoM+GMRES+MLMDA optimized, \( f = 9 \) GHz.

Fig 1. Method of moment impedance matrix \( [Z] \) depicted as an image for a five iteration Sierpinski patch antenna. If the structure is generated by an IFS, the impedance matrix has plenty of redundant information.

Fig 2. Inverse method of moment impedance matrix \( [Z]^{-1} \) depicted as an image for a five iteration Sierpinski patch antenna. The structure is similar to \( [Z] \) but here the diagonal submatrices are not equal anymore.