Study of the Koch fractal monopole in the frequency domain.

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Workpackage and task: WP1, T1.1
Security: Public
Nature: Report
Version and date: 1.0, 30-5-2002

Abstract:
The possibilities of miniaturization of fractal geometries are studied. The Koch fractal is taken as example to understand this behavior. The performance of the Koch fractal will be also compared with other non-fractal configuration (zig-zag), in order to evaluate which one is more suitable for the reduction of size of antennas.

Keyword list: Koch fractal, zig-zag, miniaturization, resonant frequency, quality factor.
RELATED WP AND TASKS (FROM THE PROJECT DESCRIPTION)


Understand better the behavior of electromagnetic fields and electric currents in fractal domains, in order to acquire guidelines for the design of fractal-shaped antennas and microwave devices.

1 INTRODUCTION

Some fractal geometries have complex, highly convoluted shapes. This property can be used to build an arbitrarily large perimeter enclosed in a finite surface or volume. Therefore, it is possible to design antennas with fractal shape that occupy the same volume as their Euclidean counterparts but much longer, and consequently, with a lower resonant frequency. The classical Koch fractal curve shown in Fig. 1 is an example of such objects where the resonant frequency of the monopole configuration decreases as the number of fractal iterations (K1, K2, K3...) increases. This particular behavior of the Koch curve was first presented in [1].

The understanding of the physic phenomena behind this behavior is the objective of the present work. For that reason the Koch fractal monopole will be compared with a zig-zag monopole (non-fractal geometry) to evaluate which configuration is more suitable for the reduction of size of antennas.

2 NUMERICAL ANALYSIS OF A KOCH FRACTAL MONOPOLE

Wire antennas can be analyzed using integral equation methods (IE) in conjunction with the well-known Method of Moments (MoM) [2]. The numerical electromagnetics code (NEC) [3] is one example of the computer codes that can deal with these problems efficiently. Nevertheless, the aforementioned codes are only valid when the wire structure is very thin, since the current is assumed to flow on the axis of the wire and the test is performed on the surface [4].

Fig. 1. Iterative construction of the fractal Koch curve.
The Koch fractal is a one-dimensional curve, but, unfortunately, the numerical simulation cannot be done using a wire model because the convoluted shape invalidates the thin-wire approximation. However, the antenna prototypes are built using printed strip technology and the strip can be modeled easily by a triangle mesh (Fig. 2), that can be accurately analyzed by MoM codes for three-dimension arbitrary objects (i.e. FIESTA code created by UPC) if enough integration points are used on each triangle.

For easier meshing, the strips can be also obtained by extrusion of the curve in the direction perpendicular to the plane containing the curve (Fig. 3). This shape, called strip-wire from now on, is a different shape than a printed antenna, but if the strip-wire is thin enough it may be a good approximation to the ideal wire. Another advantage of the strip-wire (Fig. 3) respect to the strip (Fig. 2) is that the first one allows the construction of a high number of fractal iterations since the width of the line does not affect to the bends of the geometry. In the present work, we will focus on the strip-wire configuration, the strip configuration will be analyzed in future works.

A linear monopole (K0) together with the first five iterations (K1-K5) of the Koch curve have been analyzed for comparison using the strip-wire configuration. The monopole is 6 cm height, widths of 1, 2 and 4 mm have been considered in order to evaluate the effects of the strip width.

Figures 4 and 5 show the real and imaginary part of the input impedance of the monopole obtained for the widths of 1 and 2 mm. It can be observed as the resonant frequencies become smaller as the number of fractal iterations increases. However, it must be noted that the improvement also becomes smaller with the number of iterations. At the same time, the narrower the strip-wire, the smaller the resonant frequency is. Results of the first resonant frequency in function of the number of iterations and the strip width are summarized in Table I.
Fig. 4. Input impedance for a Koch strip-wire monopole, height: 6 cm, strip width: 1 mm. The resonance frequency diminishes with the number of fractal iterations.

Fig. 5. Input impedance for a Koch strip-wire monopole, height: 6 cm, strip width: 2 mm. The resonance frequency also diminishes with the number of fractal iterations, but is slightly higher than Fig. 4.
The resonance frequency diminishes with the number of fractal iterations and increases with the strip-wire width. The resonance frequency stagnates when the number of iterations increases and the strip-wire becomes wider.

Fig. 6. When the details are small or the strip-wire becomes wider, the power signal does not follow the convoluted shape of the fractal but takes shortcuts between bends. Because of this the resonance frequency stagnates when the number of iterations grows.
The behavior is due to the coupling between curve bends: each bend radiates and receives signal, so that most of the signal power does not follow the convoluted curved paths but takes shortcuts between bends (Fig.6). The wider the strip-wire, the larger coupling between bends and the shortest way traveled by most of the signal power. This coupling agrees with the obvious fact that the resonant frequency corresponds to a monopole much shorter than the strip-wire length (table II), although of course longer than the monopole height (6 cm). The longer the curve, the more bends it has and the closer to each other they are, so the less signal follows all the meanders. As a consequence the resonance frequency stagnates as the curve length increases beyond a threshold.

3 COMPARISON WITH ZIG-ZAG MONOPOLE

The next step will be to analyze a non-fractal structure and compare the performance with the Koch fractal monopole. Let us consider a zig-zag monopole enclosing the same area than the Koch fractal (fig. 7).

Figure 8 compares the resonance frequency of both configurations in function of the strip-wire length for a width of 1 mm. Each marker in the plot corresponds to a fractal iteration in the Koch curve and a number of meanders in the zig-zag. From fig. 8 it is obvious that for the same strip-wire length the resonance frequency of the zig-zag monopole is smaller than the Koch monopole, anyway, it also stagnates as the distance between bends becomes small. This can be also understood from a geometric point of view: the zig-zag configuration distributes its bends much more separated than the Koch fractal, allowing for a longer wire (and thus a lower resonance frequency) before the coupling between bends becomes effective.

Another parameter of interest in antennas is the quality factor $Q$ which is the ratio of the radiated energy from the antenna to its stored energy. For antennas with $Q \gg 1$, the quality factor is the inverse of the fractional bandwidth. The fundamental limit of this value for small antennas was determined in [5] and is

![Fig. 4. Several configurations of the zig-zag monopole](image)
Fig. 8: Resonant frequency of Koch monopoles and zig-zag antennas of equal enclosing area as a function of strip length.

Fig. 9: Q factor of Koch monopoles (0 to 5 iterations) and zig-zag antennas with 3 and 8 meanders, compared with the fundamental limit for small antennas (h=antenna height).
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Q = \frac{1}{ka} + \frac{1}{k^3 a^3}
\]

where \( k \) is the wave number and \( a \) the radius of the smallest sphere that closes the antenna.

Figure 9 shows the Q factor of the 1mm strip Koch monopoles compared with the zig-zag antenna that has similar resonance frequency (Z3) and similar length (Z8) than the 5-iteration Koch. Although the Q factor of the Koch monopoles decreases with more fractal iterations, it converges to a minimum that is far away above the fundamental limit. Besides, it can be also seen as the quality of factor achieved for K5 is very similar to Z3, but clearly worse than Z8.

4 CONCLUSIONS

- The resonance frequency of a Koch monopole diminishes with the number of iterations, however it stagnates in a certain frequency. This is due to the coupling between close bends since the signal does not follow the path but takes shortcuts. This effect increases with the width of the strip-wire.

- From the point of view of miniaturization, the Koch fractal monopole has shown a poor performance in terms of quality factor and minimum resonance frequency achievable, in front of a zig-zag monopole of the same length enclosed in the same rectangle.

- The best quality factor achieved for both geometries is still quite far away of the fundamental for small antennas.

5 FUTURE LINES

- Validation of the previous conclusions for the case of the strip antenna of fig. 2.

- The analysis of the Koch monopole in the time domain would be worthy in order to verify that the stagnation of resonant frequency is due to the coupling between close bends.

- In the same direction, it would be interesting to establish a minimum distance of separation between bends to avoid the above mentioned coupling.

- It must not be concluded yet that fractal geometries do not provide any advantage in order to build miniature antennas. More research must be done with another fractal structures with larger fractal dimension.

6 REFERENCES

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