On the modelling of pre-fractal wire antennas

Abstract:
This report studies the accuracy of the commonly used approximations for the analysis of thin-wire antennas when applied to fractal structures. It is concluded that the thin-wire kernel of the Electric Field Integral Equation can be used only in low-iteration pre-fractals, while highly iterated ones must be modeled as an extrusion strip.

Keyword list: Numerical simulation, wire models, thin-wire approximation, pre-fractal antennas.
RELATED WP AND TASKS (FROM THE PROJECT DESCRIPTION)

WP3: Software simulation tool
   Task 3.2: Formulation of numerical methods for fractal structures
   b) Develop new approximations for thin-wire antennas along fractal curves
   c) Compare results in b) with classical numerical schemes pushed, if possible, to
      the limit of complexity afforded by the technology.

1 INTRODUCTION

Thin wire based models have a long history in numerical electromagnetics, for two
reasons: Firstly, many electromagnetical devices have a wire-like geometry. In the most
general sense, this means that they are built up of components of conducting material
that are electrically large along a given curve, and very thin ( \( << \lambda \) ) in cross section.
Examples are dipole and monopole antennas, or arrays of dipoles, Yagi-Uda antennas,
helices, etc. Secondly, under certain conditions, to be addressed in the following
section, the wire model allows for approximations that greatly facilitate the numerical
analysis.

Fractal antennas do, at first sight, comply with the definition of 'wire-like' as
formulated above, because they are built up of long and thin elements of conducting
material. However, typical of a fractal is the extreme curvedness of the elements, at all
scales of magnification (see Fig.1.1). This may undermine the applicability of the
approximations that make the strength of the thin wire model. In this report, the
applicability of the wire model to fractal antennas is evaluated.

![Fig. 1.1. Pre-fractal Koch curve.](image)

2 THIN WIRE APPROXIMATION

The thin wire approximation assumes that the conducting element can be globally
characterised by a (set of) defining curve(s) and locally by a cross-section that is

1. circular
2. has a radius \( a << \lambda \).

Such that, defining a local z-axis tangent to the defining curve:
1. The surface current density circumferential component $J_\phi$ can be neglected.

2. The surface current density axial component $J_z$ is constant along the circumference.

3. The same points 1 and 2 apply to the excitation vector field.

As a consequence, the surface current density reduces to a total current $I_z$ through the wire cross-section and the excitation vector field reduces to a potential along the defining curve.

These approximations, applied to the case of a straight thin wire, lead to the well known Pocklington equation [Balanis]:

$$
-j\varepsilon_0 E'_z = \frac{1}{2\pi} \left( k^2 + \frac{d^2}{dz^2} \right) \int I_z(z') G(R) dz'
$$

(1)

where

¡Error! Imposible crear objetos modificando códigos de campo.

(2)

is the thin wire Green's function, with

$$
R = \sqrt{(z-z')^2 + (\rho - \rho' - a\cos \phi)^2 + (a\sin \phi)^2}
$$

(3)

The extension to curved wires basically consists in sub-dividing the defining curve into segments, short enough to be approximated by straight wires, and then to apply (1) with the provision that, for non-parallel wires, $d^2 / dz^2 \rightarrow d^2 / dzdz'$.

On every straight wire segment, the current is represented by a coefficient times a basis function. Some common basis functions are shown in Fig. 2.1.

![Fig. 2.1. Examples of thin wire basis functions on a set of straight wire segments.](image-url)
3 LIMITATIONS

3.1 Segment length versus wire radius

Equation (2), the Green's function for a circumferentially constant current density, is also referred to as the full wire kernel. It has to be integrated numerically for all the mutual impedances between all wire segments in a given model (only for the auto-impedance it has an analytical solution), and this can be exceedingly time consuming. Therefore, usually, further approximations are invoked (see Report - WP3 T3.2 UPC T0+12 Wire modelling in FIESTA). However, all these further approximations have in common, that the wire radius $a$ is not only subject to $a \ll \lambda$, but also to $a \ll \Delta$, where $\Delta$ is the segment length (see Fig. 3.1).

![Fig. 3.1. $a \ll \Delta$.](image)

This obviously limits their applicability to fractal structures.

Table 3.1 shows the restrictions on the different methods addressed in [Report - WP3 T3.2 UPC T0+12 Wire modelling in FIESTA].

<table>
<thead>
<tr>
<th>Method</th>
<th>Minimum segment length $\Delta$ $(a = \text{segment radius})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>thin wire kernel</td>
<td>$8 \ a$ (1% error in current, NEC spec.)</td>
</tr>
<tr>
<td>NEC extended kernel</td>
<td>$2 \ a$ (1% error in current, NEC spec.)</td>
</tr>
<tr>
<td>Imbriale equivalent radius</td>
<td>$a$ (numerical evidence [ref], see also Fig. 3.2)</td>
</tr>
<tr>
<td>Full wire kernel</td>
<td>no restriction</td>
</tr>
</tbody>
</table>

Fig. 3.2 illustrates the difference between the equivalent radius method and the full wire kernel for $\Delta = 2 \ a$.

![Fig. 3.2. Input impedance of a straight wire, 486 segments, $\Delta = 2 \ a$.](image)
Table 3.1 indicates the limit, using the various thin wire methods, to the detail of the model structure, in other words the order (number of fractal iterations) of the pre-fractal model to be analysed. Using the full wire kernel, there is no limit with regard to the $\Delta/a$ ratio.

### 3.2 Mutually close wire segments

Another problem with highly iterated fractal structure is the occurrence of mutually very close wire segments that do not lay on the same straight line. If the mutual distance between the wire axes is of the same order as the wire radius, then the assumption that the current is constant along the circumference does not hold anymore, because of the interaction between the two segments. This is illustrated for the case of two parallel wires with a simple experiment shown in Fig. 3.3.

Of course this is true for any wire structure with bends or corners, but if the segment $\Delta/a$ is sufficiently large, and the wires do not run parallel in close proximity as they do in Fig. 3.4, the error only occurs near the end-points of the segments, and will be small compared to the total interaction term. However, in highly iterated fractal structures, every point on the wire is typically in close proximity to other non-collinear wires.

This leads to the conclusion that any thin wire model will fail for highly iterated fractals, and that the limit is not determined by $\Delta/a$ but rather by the ratio between the wire thickness and the length of the straight wire elements at the basis of the pre-fractal structure, $L \gg a$, as illustrated in Fig. 3.3.

![Fig. 3.3: Four-iteration Koch curve, with elementary wires of length L](image-url)
Another issue of importance in the application of a thin wire model to a fractal structure is the treatment of corners or wire junctions. In all the methods summed up in table 2.1 and explained in [Report - WP3 T3.2 UPC T0+12 Wire modelling in FIESTA], the segments are considered to be cylindrical tubes with flat end-faces. This means that a connection between two non-parallel segments gives rise to a situation as depicted in Fig. 3.5.

Fig. 3.4. Experiment with two parallel wires in close proximity, using triangular mesh and RWG basis functions.

3.3 Corner modeling

Another issue of importance in the application of a thin wire model to a fractal structure is the treatment of corners or wire junctions. In all the methods summed up in table 2.1 and explained in [Report - WP3 T3.2 UPC T0+12 Wire modelling in FIESTA], the segments are considered to be cylindrical tubes with flat end-faces. This means that a connection between two non-parallel segments gives rise to a situation as depicted in Fig. 3.5.

Fig. 3.5. Two non-parallel adjacent straight wire segments of finite thickness
Consequently, the surface current, which is defined on the whole cylindrical surface (excluding the endfaces, since only the axial component is considered), will, in situations like the one in Fig. 3.5, inevitably fall inside the adjacent segment.

Numerically, this effect is commonly ignored (NEC, FIESTA), which is unlikely to lead to numerical problems, since in general, only a few abscissa are needed on the wire surfaces and it is easy to prevent them from coinciding. Again, like with the problem addressed in the previous section, in the case of long thin segments \( a \ll \Delta \), the inaccuracy caused by ignoring this effect is small. However, when \( a \to \Delta \), one only needs to imagine the situation of Fig. 3.5 in the pre-fractal wire of Fig. 3.3 to see that treating the wire segments as if unaffected by adjacent segments is unrealistic.

4 NUMERICAL VALIDATION COMPARED WITH STRIPS

The surface formulation of the Electric Field Integral Equation is free from the inaccuracies that arise from the thin-wire model. Therefore, if the wire is modeled as a strip, that can be easily meshed in triangle, rooftop or quadrangular basis functions, the result is a accurate MoM discretization if enough integration points are used on each triangle [D6 Final report Task 3.1].

The strip configuration that more can model accurately a wire is a extrusion strip, obtained by extrusion of the pre-fractal curve in the direction perpendicular to the plane containing the curve (Fig. 4.1).

![Fig. 4.1. One iteration extrusion-strip Koch antenna, discretized in triangular patches.](image)

The conventional planar strip configuration has been discarded for comparison with the wire model, because the width of the strip interferes with the strip angles, while in extrusion-strips it does not. As a consequence, the geometry of a highly iterated planar strip loses the auto scaling property of pre-fractals, while extrusion-strip preserve it.

Table 4.1 shows the resonant frequency (GHz) of wire, planar-strip and extrusion-strip Koch monopoles. Height is 6cm and the strip width 1mm or 0.5mm. The wire models have a radius equal to the strip width over \( \pi \).

Two computer software codes have been used to analyze the wire model: NEC-2 [Burke] and FIESTA; with three different integral equation kernel formulations: thin-
wire approximation, extended kernel and thin-wire with equivalent wire radius [Imbrialle].

**RESONANT FREQUENCIES (GHz)**

<table>
<thead>
<tr>
<th>Software</th>
<th>NEC wire</th>
<th>NEC wire</th>
<th>FIESTA wire</th>
<th>FIESTA wire</th>
<th>FIESTA strip</th>
<th>FIESTA strip</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>thin-wire kernel</td>
<td>extended kernel</td>
<td>thin-wire kernel</td>
<td>eq. radius kernel</td>
<td>planar strip</td>
<td>extrusion strip</td>
</tr>
<tr>
<td>Radius or width</td>
<td>$0.5/\pi$ (mm)</td>
<td>$0.5/\pi$ (mm)</td>
<td>$0.5/\pi$ (mm)</td>
<td>$0.5/\pi$ (mm)</td>
<td>0.5 mm</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>K-0</td>
<td>1.202</td>
<td>1.202</td>
<td>1.238</td>
<td>1.238</td>
<td>1.195</td>
<td>1.196</td>
</tr>
<tr>
<td>K-1</td>
<td>0.987</td>
<td>0.987</td>
<td>0.993</td>
<td>0.993</td>
<td>0.988</td>
<td>0.990</td>
</tr>
<tr>
<td>K-2</td>
<td>0.866</td>
<td>0.866</td>
<td>0.864</td>
<td>0.864</td>
<td>0.858</td>
<td>0.859</td>
</tr>
<tr>
<td>K-3</td>
<td>0.799</td>
<td>0.799</td>
<td>0.785</td>
<td>0.785</td>
<td>0.787</td>
<td>0.783</td>
</tr>
<tr>
<td>K-4</td>
<td>0.773</td>
<td>0.773</td>
<td>0.772</td>
<td>0.772</td>
<td>0.763</td>
<td>0.762</td>
</tr>
<tr>
<td>K-5</td>
<td></td>
<td></td>
<td>0.782</td>
<td>0.782</td>
<td>0.758</td>
<td>0.736</td>
</tr>
</tbody>
</table>

Table 4.1: Resonant frequency of wire and strip models of the Koch antenna.

Table 4.2 shows the relative errors in the resonant frequency computation taking as a reference the results of the extrusion-strip model.

The results in tables 4.1 and 4.2 lead to the following conclusions, that corroborate the theory of the preceding sections:

- The wire models error grows rapidly after iteration K3. For three and four iterations, the pre-fractal curve segment length is respectively 2.2mm or 0.75mm, comparable to the wire diameter, and therefore the thin-wire approximation fails.
- When the thin-wire approximation fails, the error is smaller for the thinner wire. The error in the K4 1mm-strip is 3.1% and in the 0.5mm-strip 1.8%.
- The error with the equivalent radius model of Imbrialle is smaller than with the plain thin-wire approximation when the segment length becomes comparable to the wire diameter.
- For low-iteration pre-fractals, all models (wire, planar strip and extrusion strip) give the same results.
### ERROR WITH RESPECT TO EXTRUSION STRIP

<table>
<thead>
<tr>
<th>Software</th>
<th>NEC wire</th>
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<td>extrusion strip</td>
</tr>
<tr>
<td>Radius or width</td>
<td>0.5/p (mm)</td>
<td>0.5/p (mm)</td>
<td>0.5/p (mm)</td>
<td>0.5/p (mm)</td>
<td>0.5 mm</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>K-0</td>
<td>0.53%</td>
<td>0.53%</td>
<td>3.54%</td>
<td>3.54%</td>
<td>-0.06%</td>
<td>0.00%</td>
</tr>
<tr>
<td>K-1</td>
<td>-0.27%</td>
<td>-0.27%</td>
<td>0.31%</td>
<td>0.30%</td>
<td>-0.19%</td>
<td>0.00%</td>
</tr>
<tr>
<td>K-2</td>
<td>0.78%</td>
<td>0.79%</td>
<td>0.61%</td>
<td>0.49%</td>
<td>-0.12%</td>
<td>0.00%</td>
</tr>
<tr>
<td>K-3</td>
<td>2.02%</td>
<td>2.03%</td>
<td>0.18%</td>
<td>-0.04%</td>
<td>0.46%</td>
<td>0.00%</td>
</tr>
<tr>
<td>K-4</td>
<td>3.04%</td>
<td>3.06%</td>
<td>2.92%</td>
<td>1.76%</td>
<td>1.59%</td>
<td>0.00%</td>
</tr>
<tr>
<td>K-5</td>
<td></td>
<td></td>
<td>6.28%</td>
<td>2.93%</td>
<td></td>
<td>0.00%</td>
</tr>
</tbody>
</table>

<table>
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<th>NEC wire</th>
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</tr>
<tr>
<td>Radius or width</td>
<td>1/p (mm)</td>
<td>1/p (mm)</td>
<td>1/p (mm)</td>
<td>1/p (mm)</td>
<td>1 mm</td>
<td>1 mm</td>
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<tr>
<td>K-0</td>
<td>0.47%</td>
<td>0.47%</td>
<td>3.69%</td>
<td>3.70%</td>
<td>-0.27%</td>
<td>0.00%</td>
</tr>
<tr>
<td>K-1</td>
<td>-0.37%</td>
<td>-0.33%</td>
<td>-0.33%</td>
<td>0.42%</td>
<td>0.38%</td>
<td>-0.27%</td>
</tr>
<tr>
<td>K-2</td>
<td>1.05%</td>
<td>1.08%</td>
<td>-0.60%</td>
<td>-0.69%</td>
<td>-0.07%</td>
<td>0.00%</td>
</tr>
<tr>
<td>K-3</td>
<td>2.55%</td>
<td>2.49%</td>
<td>1.40%</td>
<td>0.80%</td>
<td>0.82%</td>
<td>0.00%</td>
</tr>
<tr>
<td>K-4</td>
<td>4.00%</td>
<td>4.11%</td>
<td>8.48%</td>
<td>3.10%</td>
<td>2.70%</td>
<td>0.00%</td>
</tr>
<tr>
<td>K-5</td>
<td></td>
<td></td>
<td>11.35%</td>
<td>-1.74%</td>
<td></td>
<td>0.00%</td>
</tr>
</tbody>
</table>

Table 4.2: Error in the resonant frequency computation with respect to the extrusion-strip model.
5 CONCLUSIONS

Based on the findings in section 3 and 4, the following conclusions can be drawn regarding the use of the thin wire model for the analysis of pre-fractal antennas:

- An important parameter in thin straight wire models is the ratio of segment length and segment radius ($\Delta/a$). For $\Delta/a>1$, fast approximative methods can be used, for $\Delta/a<1$, one needs to revert to the more computationally demanding full wire kernel.

- However, in pre-fractal structures the occurrence of many sharp bends (corners) limits this ratio to $\Delta/a>>1$; and the error introduced by mutually close non-collinear segments restricts the minimum length $L$ of the pre-fractal model wire segments to $L>>a$. In both cases the assumption of constant current along the wire circumference fails.

- Summarising the above: The thin wire model can only be used with confidence for modelling pre-fractal structures of a sufficiently low order (iteration) that $L>>a$. Then, the first criterion, $\Delta/a>>1$, can easily be fulfilled by using one segment per fractal building block.

- The best way to accurately model highly iterated pre-fractal curves is a extrusion-strip model rather than a planar strip or a thin wire. For that reason, most of the simulations in WP3 aimed at drawing conclusions for WP1 will be made with extrusion-strip models.

- However, wire models are still useful for analysing low-order pre-fractals, and therefore NEC, FIESTA and the time-domain DOTIG code will be extensively used in this project.
BIBLIOGRAPHY


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